

# Delaware Content Standards Grade Band Extensions Mathematics





## Acknowledgements

Development of the Delaware Content Standards Grade Band Extensions was a collaborative effort between the Delaware Department of Education and the Center for Disabilities Studies at the University of Delaware. Committee panels, comprised of special and general educators throughout the state of Delaware, worked together to create the extensions. Much appreciation to:

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Special thanks to Martha Toomey, Brian M. Touchette, and the staff of Exceptional Children Resources at the Delaware Department of Education for their funding and support. With gratitude to the many family and community members, school administrators, and university faculty who provided valuable input and guidance through advisory panels. The Delaware Content Standards Grade Band Extensions would not be possible without the support of all contributors who worked tirelessly to create a guiding tool to support access to the general education curriculum for all students.

# Table of Contents

|  |    |
|--|----|
| Introduction .....                                   | 5  |
| <b>Navigating the Grade Band Extensions</b> .....    | 6  |
| <b>Grade Band K-2</b>                                |    |
| Counting and Cardinality.....                        | 9  |
| Operations and Algebraic Thinking.....               | 11 |
| Number and Operations in Base Ten .....              | 12 |
| Measurement and Data.....                            | 14 |
| Geometry .....                                       | 16 |
| <b>Grade Band 3-5</b>                                |    |
| Operations and Algebraic Thinking.....               | 19 |
| Number and Operations in Base Ten .....              | 20 |
| Number and Operations—Fractions .....                | 21 |
| Measurement and Data.....                            | 23 |
| Geometry .....                                       | 25 |
| <b>Grade Band 6-8</b>                                |    |
| Ratios and Proportional Relationships.....           | 29 |
| The Number System .....                              | 30 |
| Expressions and Equations .....                      | 31 |
| Functions.....                                       | 33 |
| Geometry .....                                       | 34 |
| Statistics and Probability.....                      | 36 |
| <b>Grade Band HS</b>                                 |    |
| Algebra  |    |
| Seeing Structure in Expressions .....                | 39 |
| Creating Equations .....                             | 40 |
| Reasoning with Equations and Inequalities.....       | 41 |
| Geometry   |    |
| Congruence .....                                     | 42 |
| Similarity, Right Triangles, and Trigonometry .....  | 43 |
| Circles.....   | 44 |
| Geometric Measurement and Dimension.....             | 45 |
| Modeling with Geometry.....                          | 46 |
| Statistics and Probability                           |    |
| Interpreting Categorical and Quantitative Data ..... | 47 |
| Making Inferences and Justifying Conclusions .....   | 48 |

## Introduction

In August 2010, the state of Delaware adopted the Common Core State Standards (CCSS) for English Language Arts and Mathematics. The CCSS are now part of the Delaware Content Standards. Recognizing the need to make the content standards accessible for all students, the Delaware Department of Education in collaboration with the Center for Disabilities Studies at the University of Delaware developed the Delaware Grade Band Extensions.

### Grade Band Extensions

The extensions are designed to assist teachers in providing access to the general education curriculum for students with significant intellectual disabilities. Students receiving instruction based on the grade band extensions total approximately 1% of Delaware's student population and are assessed through the Delaware Comprehensive Assessment System Alternate Assessment (DCAS-Alt1).

Standards (academic skills) from the CCSS were identified within the following grade bands: K-2, 3-5, 6-8, and High School. For each standard, three levels of extensions were created from highest to lowest complexity. Extensions are meant to provide a continuum of entry points related to the English Language Arts and Mathematics standards. Following the full text of the standard, the extensions are organized as follows:

- Extension 1—Most complex application of the standard
- Extension 2—Less complex application of the standard
- Extension 3—Least complex application of the standard

Within each grade band, standards were selected from the highest grade. If a concept was not represented within the highest grade, a standard that reflects the concept was included from a lower grade. Some standards within specific grade bands were not extended due to the complexity of the skills and are not included in this document.

*Please note, students should not be categorized according to a particular extension level. Instead, instruction should target extensions appropriate to individual strengths which may vary across standards.*

### Purpose

The extensions will be used by school personnel to plan and implement lessons based on academic standards. Extensions will assist special educators in planning academic activities aligned to the state standards. In planning academic activities, teachers must consider incorporation of non-academic skills necessary for student success such as communication, self-determination, gross/fine motor, and social skills.

Additionally, the extensions will be the basis of the DCAS-Alt1. Students will participate in DCAS-Alt1 testing in the fall and spring of each school year. Test items will be aligned to the extensions. Therefore, it is imperative that students receive instruction aligned to the extensions throughout the school year.

### Considerations for Use

The Delaware Grade Band Extensions do not specify individual accommodations or supports that may be necessary for students to access the curriculum. When designing lessons based on the extensions, teachers should consider the unique learning needs of each student and employ the necessary accommodations. According to the CCSS *Application to Students with Disabilities* document (CCSSO 2010),

These supports and accommodations should ensure that students receive access to multiple means of learning and opportunities to demonstrate knowledge, but retain the rigor and high expectations of the Common Core State Standards.

In addition to considering accommodations, teachers should reference the complete CCSS document for grade-specific standards, text and writing exemplars, and suggested texts. The Delaware Grade Band Extensions are not meant to replace the Delaware Content Standards but to be used as a companion document. To view the Delaware Content Standards which include the CCSS, visit the Delaware Department of Education website at [www.doe.k12.de.us](http://www.doe.k12.de.us).

## Navigating the Delaware Grade Band Extensions

The document is divided by **grade bands**: K-2, 3-5, 6-8, HS.

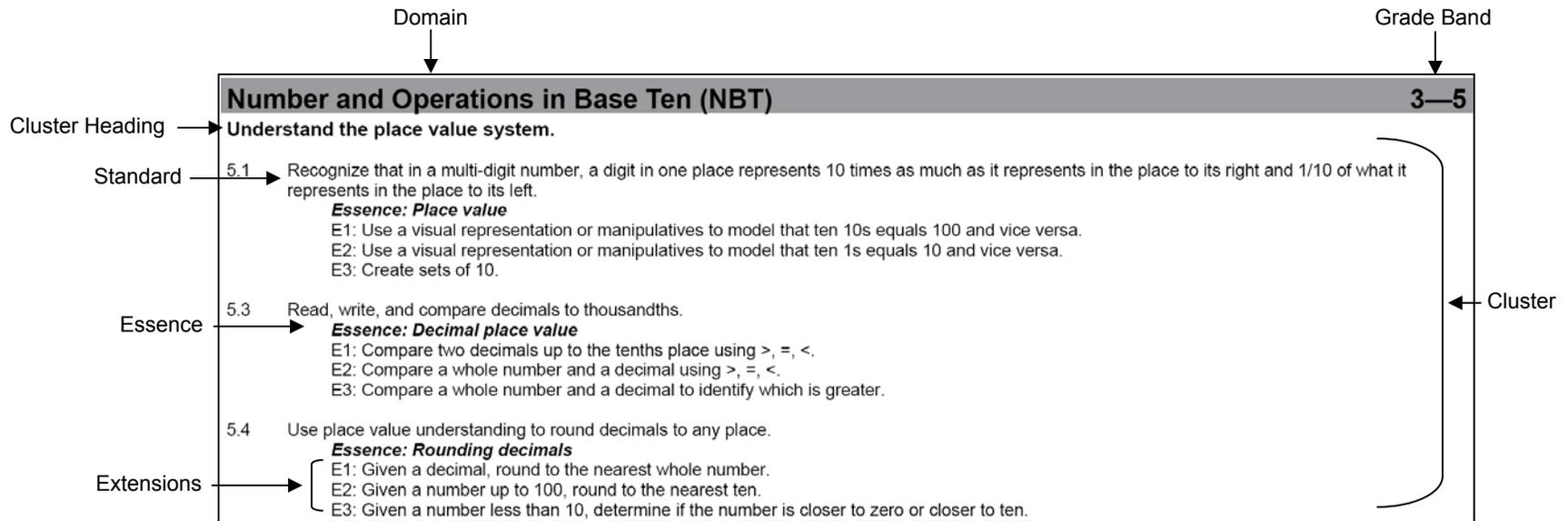
**Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.

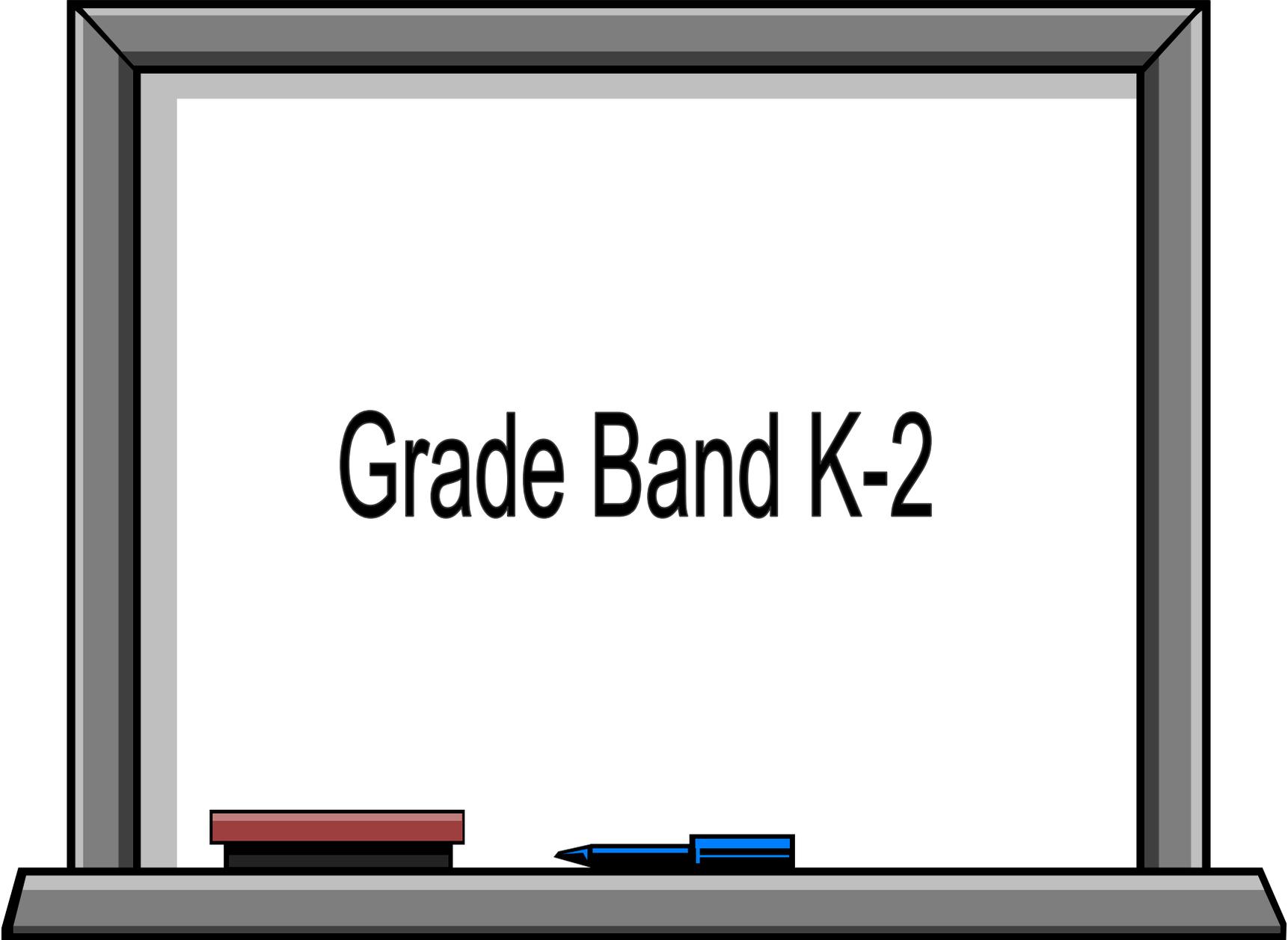
**Clusters** are groups of related standards. Note that standards from different clusters may sometimes be closely related because mathematics is a connected subject.

**Standards** define what students should understand and be able to do.

**Essence** is the main idea of the standard.

**Extensions** are entry points to the standard.





Grade Band K-2



### Know number names and the count sequence.

- K.1 Count to 100 by ones and by tens.  
**Essence: Counting**  
E1: Count up to 50 by 1s or 10s.  
E2: Count up to 20 by 1s.  
E3: Using a model, count up to 10 by 1s.
- K.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).  
**Essence: Count up from a given number**  
E1: Given a starting number (11-50), identify what number comes next.  
E2: Given a starting number (2-10), identify what number comes next.  
E3: Given a starting number (2-10), identify what number comes next from a choice of two numbers.
- K.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).  
**Essence: Represent objects with written numerals**  
E1: Write the correct number for objects up to 15.  
E2: Match objects to the correct number up to 15.  
E3: Match objects to the correct number up to 5.

### Count to tell the number of objects.

- K.4 Understand the relationship between numbers and quantities; connect counting to cardinality.  
**Essence: One-to-one correspondence and concept of one more**  
E1: Using one-to-one correspondence, count a given number of objects and identify the next number within the range of 1-20.  
E2: Given a number of objects, identify the number that corresponds to that quantity within the range of 1-20.  
E3: Count with one-to-one correspondence within the range of 1-20.
- K.5 Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.  
**Essence: Count to answer “how many?”**  
E1: Given a specified number from 1-10, count the correct number of objects.  
E2: Given a rectangular array or line containing up to 10 objects, count the total number of objects.  
E3: Given a line of up to five objects, count the total number of objects.

**Compare numbers.**

K.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.

***Essence: Compare groups of objects***

E1: Given two groups of objects, identify which group is greater (exclude equal sets).

E2: Given two groups of objects, identify whether the groups are equal to each other.

E3: Given two groups of objects arranged in a line side by side, identify whether the groups are equal to each other.

K.7 Compare two numbers between 1 and 10 presented as written numerals.

***Essence: Compare written numerals***

E1: Compare two written numerals 1-5 without a model.

E2: Compare two written numerals 1-10 given a written model.

E3: Compare written numerals 1-5 given an object model.

### Represent and solve problems involving addition and subtraction.

- 2.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

**Essence: Solve problems involving addition and subtraction**

- E1: Given the number sentence for a one-step word problem involving numbers less than 20, solve using addition or subtraction.  
E2: Given an object model for a one-step word problem involving numbers less than 10, solve using addition or subtraction.  
E3: Demonstrate “putting together” or “more” as addition and “taking away” or “less” as subtraction.

### Add and subtract within 20.

- 2.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

**Essence: Add and subtract**

- E1: Add and subtract within 15.  
E2: Add or subtract within 10.  
E3: Identify “putting together” or “more” as addition and “taking away” or “less” as subtraction.

### Work with equal groups of objects to gain foundations for multiplication.

- 2.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

**Essence: Pair objects to determine odd or even**

- E1: Identify groups as odd or even.  
E2: Identify groups as odd or even by pairing objects.  
E3: Group objects together by 2s (pairs).

- 2.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

**Essence: Find the total number of objects using repeated addition**

- E1: Add numbers in an array with up to four rows and four columns.  
E2: Add numbers or objects in an array with up to three rows and three columns.  
E3: Arrange objects into groups to show repeated addition.

### Understand place value.

- 2.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.  
**Essence: Place value**  
E1: Given a two-digit number, break apart the number into tens and ones ( $18 = 10 + 8$ ).  
E2: Given a two-digit number, identify the number in the tens or ones place.  
E3: Identify an object representation for a two-digit number.
- 2.2 Count within 1000; skip-count by 5s, 10s, and 100s.  
**Essence: Skip counting**  
E1: Count by 5s up to 100.  
E2: Count by 100s up to 1000.  
E3: Count by 10s up to 100.
- 2.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.  
**Essence: Number representations**  
E1: Given a two-digit number written in standard form, identify the correct expanded form or number name ( $64 = 60 + 4 = \text{sixty-four}$ ).  
E2: Given a visual representation of a two-digit number, identify the correct expanded form.  
E3: Given an object representation for a two-digit number, identify the number.
- 2.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.  
**Essence: Compare number values**  
E1: Compare two-digit numbers using symbols or words.  
E2: Compare one-digit numbers using symbols or words.  
E3: Given an object model, compare one-digit numbers using words.

Use place value understanding and properties of operations to add and subtract.

- 2.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.  
**Essence: Add and subtract**  
E1: Add or subtract within 50.  
E2: Add or subtract within 20.  
E3: Add one-digit numbers.
- 2.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.  
**Essence: Add two-digit numbers**  
E1: Add two two-digit numbers without carrying.  
E2: Add two two-digit numbers with 0s in the one's place.  
E3: Add one-digit numbers.
- 2.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.  
**Essence: Line up digits according to place value to add or subtract**  
E1: Given a two-digit and one-digit horizontal problem with no carrying, set up the problem by lining up place values vertically and solve using addition or subtraction.  
E2: Given a two-digit and one-digit vertical problem with no carrying, solve using addition or subtraction.  
E3: Given a problem with one-digit numbers, solve using addition or subtraction.
- 2.8 Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.  
**Essence: Mentally add or subtract 10 or 100 to/from a given number**  
E1: Mentally add and subtract 10 to/from a given number up to 100.  
E2: Add 10 to a given number up to 100.  
E3: Given a visual or object model, add 1 to a given number up to 10.
- 2.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.  
**Essence: Addition and subtraction strategies**  
E1: Given a subtraction problem, create a model to represent how to solve the problem.  
E2: Given an addition problem, create a model to represent how to solve the problem.  
E3: Given an addition problem and two models, choose which model represents the problem.

**Measure and estimate lengths in standard units.**

- 2.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.  
**Essence: Measure length with an appropriate tool**  
E1: Measure an object with a given tool such as a ruler, yardstick, meter stick, or measuring tape.  
E2: Given two measuring tools, identify which is closer to the length of an object.  
E3: Order objects by length.
- 2.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.  
**Essence: Compare different units of measure**  
E1: Measure the length of an object with two different standard or nonstandard tools.  
E2: Measure an object with a standard or nonstandard tool.  
E3: Compare two units of measure and identify as longer, shorter, or same length.
- 2.3 Estimate lengths using units of inches, feet, centimeters, and meters.  
**Essence: Estimate length**  
E1: Estimate length using inches or feet.  
E2: Estimate length using non-standard units.  
E3: Given two rulers (standard or non-standard) of different lengths, choose the ruler that best represents the length of the object.
- 2.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.  
**Essence: Compare length of objects**  
E1: Measure two objects using standard units and compare their lengths.  
E2: Measure two different objects using standard or nonstandard tools.  
E3: Compare the length of two objects and identify as longer, shorter, or same length.

**Relate addition and subtraction to length.**

- 2.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.  
**Essence: Solve for an unknown length in a word problem**  
E1: Create a model representing the key information from a word problem involving addition or subtraction of lengths and solve.  
E2: Given a model of a measurement word problem, use addition or subtraction to solve.  
E3: Identify the longer and shorter length within a word problem.
- 2.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.  
**Essence: Represent whole numbers on a number line and use to add or subtract**  
E1: Add or subtract using a number line.  
E2: Demonstrate that moving forward is addition and moving backwards is subtraction.  
E3: Identify the placement of numbers on a number line.

**Work with time and money.**

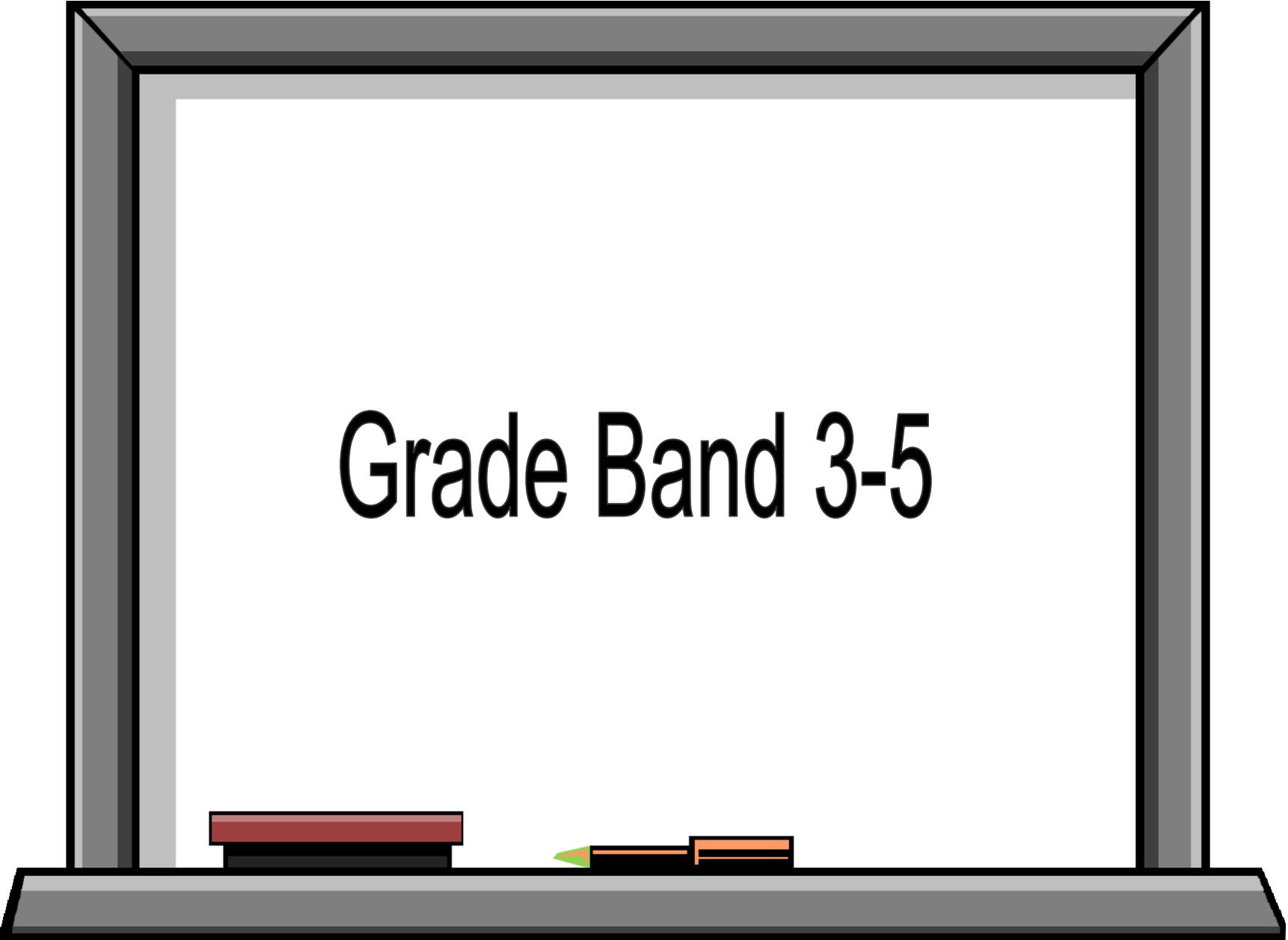
- 2.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.  
**Essence: Tell time**  
E1: Tell time to the nearest hour.  
E2: Use a schedule to determine the order of events.  
E3: Connect time to an event (activities that happen in the morning [am] and in the evening [pm]).
- 2.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*  
**Essence: Solve word problems involving money**  
E1: Identify the value of a combination of coins within a word problem.  
E2: Identify coins and/or match to their values.  
E3: Match like coins.

**Represent and interpret data.**

- 2.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.  
**Essence: Gather and represent measurement data using a line plot**  
E1: Measure length of object(s) to the nearest whole unit and locate length on a line plot.  
E2: Given points of measurement data, identify location of lengths on a line plot or number line.  
E3: Identify the placement of numbers on a number line.
- 2.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.  
**Essence: Represent data on a graph**  
E1: Given data, create a bar or picture graph and answer questions about the data.  
E2: Given data, create a bar or picture graph.  
E3: Given a picture or bar graph, answer literal questions about the data.

## Reason with shapes and their attributes.

- 2.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.  
**Essence: Identify shapes**  
E1: Name shapes.  
E2: Sort shapes.  
E3: Match shapes.
- 2.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.  
**Essence: A rectangle can be partitioned into same-size squares**  
E1: Cover a rectangle with smaller same-size square pieces and count the number of pieces.  
E2: Cover a rectangle with smaller same-size square pieces.  
E3: Identify a rectangle or square from other shapes.
- 2.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.  
**Essence: A whole can be partitioned into sets of equal parts**  
E1: Cover a shape with two or more different sets of same-size pieces (halves, thirds, quarters) and count the number of smaller pieces needed to cover the whole.  
E2: Cover a shape with two or more different sets of same-size pieces (halves, quarters).  
E3: Identify circles and rectangles from other shapes.



Grade Band 3-5



**Write and interpret numerical expressions.**

- 5.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

**Essence: Use grouping symbols to evaluate expressions**

E1: Evaluate addition and subtraction expressions using grouping symbols.

E2: Identify the operation(s) needed to evaluate expressions.

E3: Use manipulatives to add and subtract numbers less than 20.

- 5.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation “add 8 and 7, then multiply by 2” as  $2 \times (8 + 7)$ . Recognize that  $3 \times (18932 + 921)$  is three times as large as  $18932 + 921$ , without having to calculate the indicated sum or product.*

**Essence: Interpret numerical expressions using grouping symbols**

E1: Identify which operation comes first when a calculation requires up to two operations.

E2: Use a visual representation or manipulatives to model a problem that requires two operations.

E3: Use manipulatives to model a problem that requires one operation.

**Analyze patterns and relationships.**

- 5.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

**Essence: Generate patterns**

E1: Given a rule, create a sequence with up to five points/numbers.

E2: Continue a sequence of numbers with a given rule.

E3: Continue a non-numeric pattern.

## Understand the place value system.

- 5.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

**Essence: Place value**

- E1: Use a visual representation or manipulatives to model that ten 10s equals 100 and vice versa.  
 E2: Use a visual representation or manipulatives to model that ten 1s equals 10 and vice versa.  
 E3: Create sets of 10.

- 5.3 Read, write, and compare decimals to thousandths.

**Essence: Decimal place value**

- E1: Compare two decimals up to the tenths place using  $>$ ,  $=$ ,  $<$ .  
 E2: Compare a whole number and a decimal using  $>$ ,  $=$ ,  $<$ .  
 E3: Compare a whole number and a decimal to identify which is greater.

- 5.4 Use place value understanding to round decimals to any place.

**Essence: Rounding decimals**

- E1: Given a decimal, round to the nearest whole number.  
 E2: Given a number up to 100, round to the nearest ten.  
 E3: Given a number less than 10, determine if the number is closer to zero or closer to ten.

## Perform operations with multi-digit whole numbers and with decimals to hundredths.

- 5.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

**Essence: Multiply whole numbers**

- E1: Repeated addition of a two-digit and single-digit number ( $20 \times 4 = 20 + 20 + 20 + 20$ ).  
 E2: Repeated addition of single-digit numbers ( $3 \times 5 = 3 + 3 + 3 + 3 + 3$ ).  
 E3: Identify a model representing repeated addition of single-digit numbers from a choice of two or more.

- 5.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**Essence: Divide whole numbers**

- E1: Given a two-digit number, divide by a one-digit number with no remainder.  
 E2: Given a set of objects up to 20, divide objects into equal groups.  
 E3: Given a set of objects up to 10, divide into equal groups.

- 5.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

**Essence: Decimal operations**

- E1: Solve decimal problems involving subtraction.  
 E2: Solve decimal problems involving addition.  
 E3: Round a decimal to the nearest whole number.

## Use equivalent fractions as a strategy to add and subtract fractions.

- 5.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,  $2/3 + 5/4 = 8/12 + 15/12 = 23/12$ . (In general,  $a/b + c/d = (ad + bc)/bd$ .)*  
**Essence: Add and subtract fractions**  
 E1: Add or subtract fractions with like denominators.  
 E2: Given a visual or object model, add or subtract fractions with like denominators.  
 E3: Given an object model with one fraction displayed, add a fraction with a like denominator.
- 5.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result  $2/5 + 1/2 = 3/7$ , by observing that  $3/7 < 1/2$ .*  
**Essence: Fraction word problems**  
 E1: Solve addition or subtraction word problems involving fractions with like denominators.  
 E2: Given a visual or object model, solve addition or subtraction word problems involving fractions with like denominators.  
 E3: Given a visual or object model of a fraction and the denominator, identify the numerator.

## Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- 5.3 Interpret a fraction as division of the numerator by the denominator ( $a/b = a \div b$ ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret  $3/4$  as the result of dividing 3 by 4, noting that  $3/4$  multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size  $3/4$ . If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*  
**Essence: Fractions represent division problems**  
 E1: Given a fraction, identify the numerator and denominator.  
 E2: Given a visual or object model, construct a fraction.  
 E3: Given a visual or object model of a fraction and the denominator, identify the numerator.
- 5.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.  
**Essence: Multiply fractions**  
 E1: Multiply a fraction by a whole number using repeated addition ( $1/2 \times 3 = 1/2 + 1/2 + 1/2$ ).  
 E2: Using a visual or object model, multiply a fraction by a whole number using repeated addition.  
 E3: Identify a model representing multiplication of a fraction by a whole number from a choice of two.
- 5.5 Interpret multiplication as scaling (resizing).  
**Essence: Multiplying by a whole number produces a bigger product; multiplying by a fraction produces a smaller product**  
 E1: Multiply a fraction by a whole number and then compare the product to the original whole number ( $1/2 \times 4 = 2$  and 2 is smaller than 4).  
 E2: Compare fractions using  $>$ ,  $=$ ,  $<$ .  
 E3: Given a visual or object model, compare two fractions and identify which is bigger or smaller.

**Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (continued)**

- 5.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

***Essence: Multiplication word problems involving fractions***

E1: Solve multiplication word problems involving a fraction and a whole number.

E2: Given a visual or object model, solve multiplication word problems involving a fraction and a whole number.

E3: Given a visual or object model of a fraction and its denominator, identify the numerator.

- 5.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

***Essence: Divide whole numbers by fractions***

E1: Given a visual or object model, divide a whole number by a fraction.

E2: Given a visual or object model, divide a whole number by  $\frac{1}{4}$  or  $\frac{1}{2}$ .

E3: Given a set of objects, divide the set into half and count the pieces.

**Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**

- 3.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

**Essence: Tell time**

E1: Tell time to the nearest quarter or half hour.

E2: Tell time to the nearest hour.

E3: Use a schedule to determine the order of events.

**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**

- 4.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

**Essence: Solve real world problems involving time and money**

E1: Solve word problems involving elapsed time or amount of money necessary for a purchase.

E2: Estimate the amount of elapsed time or the amount of money necessary for a purchase.

E3: Identify the appropriate interval of time or amount of money to solve a problem.

**Convert like measurement units within a given measurement system.**

- 5.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

**Essence: Equivalent measurement**

E1: Measure an object using two measurements and compare the measures (<, =, >).

E2: Measure an object using two measurements.

E3: Order measures from smallest to biggest or shortest to longest and vice versa.

**Represent and interpret data.**

- 5.2 Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

**Essence: Organize and represent data**

E1: Create a line plot using  $\frac{1}{2}$  and whole numbers.

E2: Create a line plot using whole numbers.

E3: Match data points to a given line plot or number line.

**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

5.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

**Essence: Properties of volume**

E1: Determine how much a three-dimensional figure holds by filling with specified units (cup, ounce, liter).

E2: Fill a three-dimensional figure to a specified volume (cup, ounce, liter).

E3: Identify a three-dimensional figure as full or empty.

5.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

**Essence: Measure volume**

E1: Choose the appropriate units (cups, ounces, liters, gallons) to measure volume of a selected container.

E2: Find volume by counting unit cubes or improvised units.

E3: Fill containers to capacity.

5.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

**Essence: Relate volume to addition and multiplication**

E1: Find the total volume of two combined measures.

E2: Choose an appropriate container to hold combined volumes of two containers.

E3: Indicate if more or less is needed to fill containers to a specified line of measurement.

**Graph points on the coordinate plane to solve real-world and mathematical problems.**

- 5.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

**Essence: Plot coordinates on a graph**

- E1: Given a coordinate system, plot points on the plane.  
E2: Given a coordinate system, identify the coordinates of a point.  
E3: Locate numbers on a number line.

- 5.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

**Essence: Plot coordinates on a graph**

- E1: Given a coordinate system, plot points on the plane.  
E2: Given a coordinate system, identify the coordinates of a point.  
E3: Locate numbers on a number line.

**Classify two-dimensional figures into categories based on their properties.**

- 5.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

**Essence: Categories of shapes have similar attributes**

- E1: Label attributes of shapes.  
E2: Sort shapes according to attributes.  
E3: Match shapes.

- 5.4 Classify two-dimensional figures in a hierarchy based on properties.

**Essence: Classify shapes**

- E1: Sort shapes by multiple attributes and label the attributes.  
E2: Sort shapes by multiple attributes.  
E3: Sort shapes according to a given attribute.



Grade Band 6-8





## Ratios and Proportional Relationships (RP)

6—8

Analyze proportional relationships and use them to solve real-world and mathematical problems.

- 7.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks  $\frac{1}{2}$  mile in each  $\frac{1}{4}$  hour, compute the unit rate as the complex fraction  $\frac{1}{2} \div \frac{1}{4}$  miles per hour, equivalently 2 miles per hour.*  
**Essence: Create and solve ratios**  
E1: Given a visual model, identify ratios involving fractions.  
E2: Given a visual model or manipulative, identify ratios involving whole numbers.  
E3: Given a manipulative, identify the units to be compared.
- 7.2 Recognize and represent proportional relationships between quantities.  
**Essence: Represent proportional relationships**  
E1: Identify if a graph represents a proportional relationship.  
E2: Given coordinate pairs involving whole numbers, identify the rule.  
E3: Given a rule, continue a sequence of whole numbers.
- 7.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*  
**Essence: Use ratios to solve real world problems**  
E1: Given a ratio table involving whole numbers, identify the rule and fill in a missing value.  
E2: Given a ratio table involving whole numbers, identify the rule.  
E3: Given a rule, continue a sequence of whole numbers.

**Know that there are numbers that are not rational, and approximate them by rational numbers.**

- 8.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

**Essence: Identify rational and irrational numbers**

E1: Identify and extend the pattern in a rational decimal.

E2: Identify the pattern in a rational decimal.

E3: Given a rule, continue a sequence of whole numbers.

- 8.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ). *For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

**Essence: Estimate the values of numbers**

E1: Given decimals, identify corresponding points on a number line and compare.

E2: Round decimals to the nearest whole number and identify the corresponding points on a number line.

E3: Given whole numbers, identify the corresponding points on a number line.

## Work with radicals and integer exponents.

- 8.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,  $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .*

**Essence: Know how to evaluate numerical expressions that contain exponents**

E1: Express single-digit exponents in expanded form ( $3^3 = 3 \times 3 \times 3$ ).

E2: Identify a model representing repeated multiplication (exponents in expanded form).

E3: Identify a model representing repeated addition (multiplication) of single-digit numbers.

- 8.2 Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that  $\sqrt{2}$  is irrational.

**Essence: Evaluate square root and cube root of perfect squares and cubes**

E1: Recognize perfect squares up to 25.

E2: Create a representation of a perfect square.

E3: Given a visual or manipulative model, select the perfect square.

- 8.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as  $3 \times 10^8$  and the population of the world as  $7 \times 10^9$ , and determine that the world population is more than 20 times larger.*

**Essence: Understand the powers of 10**

E1: Given two numbers expressed as single digits times an integer power of 10, identify as  $>$ ,  $=$ ,  $<$ .

E2: Match a number with its scientific notation form (single digit times an integer power of 10).

E3: Compare numbers greater than 100 in standard form.

- 8.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

**Essence: Scientific notation**

E1: Simplify a single digit times an integer power of 10 ( $6 \times 10^3 = 6,000$ ).

E2: Match a number with its scientific notation form (single digit times an integer power of 10).

E3: Compare numbers greater than 100 in standard form.

## Understand the connections between proportional relationships, lines, and linear equations.

- 8.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

**Essence: Graph and compare slope**

E1: When given similar representations, identify the greater slope (steeper line has the greater slope).

E2: Identify if a line has positive, negative, or no slope.

E3: Identify directionality of lines (going up, going down).

**Understand the connections between proportional relationships, lines, and linear equations. (continued)**

- 8.6 Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .

**Essence: Define slope and y-intercept**

E1: Identify the coordinates for points on a line.

E2: Given a line on a graph, identify the y-intercept.

E3: Identify the x and y axes.

**Analyze and solve linear equations and pairs of simultaneous linear equations.**

- 8.7 Solve linear equations in one variable.

**Essence: Solve linear equations**

E1: Solve a one-step equation with positive numbers involving addition and subtraction only ( $y + 3 = 5$ ).

E2: Given a one-step equation, identify the operation needed to solve (the inverse operation).

E3: Solve for an unknown number within a number sentence ( $5 + x = 8$ ).

- 8.8 Analyze and solve pairs of simultaneous linear equations.

**Essence: Solve pairs of linear equations**

E1: Given a graph with two lines, compare the slopes of the lines (line a is steeper than line b).

E2: Given a graph with two lines, identify the coordinates of the point of intersection.

E3: Given a graph with two lines, identify the point of intersection.

## Define, evaluate, and compare functions.

- 8.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.  
**Essence: Specific input will yield specific output**  
 E1: Given a set of at least three ordered pairs, identify the next ordered pair in the sequence.  
 E2: Identify the rule of a numeric pattern.  
 E3: Given the rule, continue a numeric pattern.
- 8.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*  
**Essence: Compare/contrast two different input/output relationships**  
 E1: Describe the similarities/differences between a set of repeated patterns.  
 E2: Identify similar sets of repeated patterns presented numerically in tables.  
 E3: Identify similar sets of repeated patterns represented graphically.
- 8.3 Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*  
**Essence: Equations of linear and non-linear functions**  
 E1: Describe the similarities/differences of linear or non-linear functions.  
 E2: Classify graphs of functions as linear or non-linear.  
 E3: Identify the directionality (going up, going down) of a line on a graph.

## Use functions to model relationships between quantities.

- 8.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.  
**Essence: Construct a linear graph using a table or equation**  
 E1: Given a graph, plot multiple coordinates.  
 E2: Identify the coordinates of a given points.  
 E3: Identify the directionality (going up, going down) of a line on a graph.
- 8.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.  
**Essence: Construct a linear graph as described verbally**  
 E1: Find multiple sets of coordinate pairs on a graph plot based on verbal directions.  
 E2: Find one set of coordinate pairs on a graph plot based on verbal directions.  
 E3: Find numbers on a line diagram/number line based on verbal directions.

Understand congruence and similarity using physical models, transparencies, or geometry software.

- 8.1 Verify experimentally the properties of rotations, reflections, and translations:  
**Essence: Demonstrate rotations (turns), reflections (flips), and translations (slides)**  
 E1: Manipulate multiple shapes to fit matching spaces.  
 E2: Match shapes in different orientations.  
 E3: Manipulate shapes to fit matching spaces.
- 8.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.  
**Essence: Properties of shapes stay the same regardless of orientation**  
 E1: Manipulate multiple shapes to fit matching spaces.  
 E2: Match a shapes in different orientations.  
 E3: Manipulate shapes to fit the matching space.
- 8.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.  
**Essence: Describe the manipulation or resizing of geometric figures**  
 E1: Identify the orientation of a figure related to another figure.  
 E2: Demonstrate concepts of size and directionality (top, bottom, right, left, flips, turns).  
 E3: Match shapes in different orientations and sizes.
- 8.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.  
**Essence: Properties of shapes stay the same regardless of orientation or size**  
 E1: Describe the steps needed to match shapes in different orientations or sizes.  
 E2: Match shapes in different orientations and sizes.  
 E3: Manipulate shapes to fit matching spaces.
- 8.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*  
**Essence: Properties of angles**  
 E1: Arrange angles in order from smallest to largest and vice versa.  
 E2: Sort angles into predetermined categories.  
 E3: Match identical angles.

**Understand and apply the Pythagorean Theorem.**

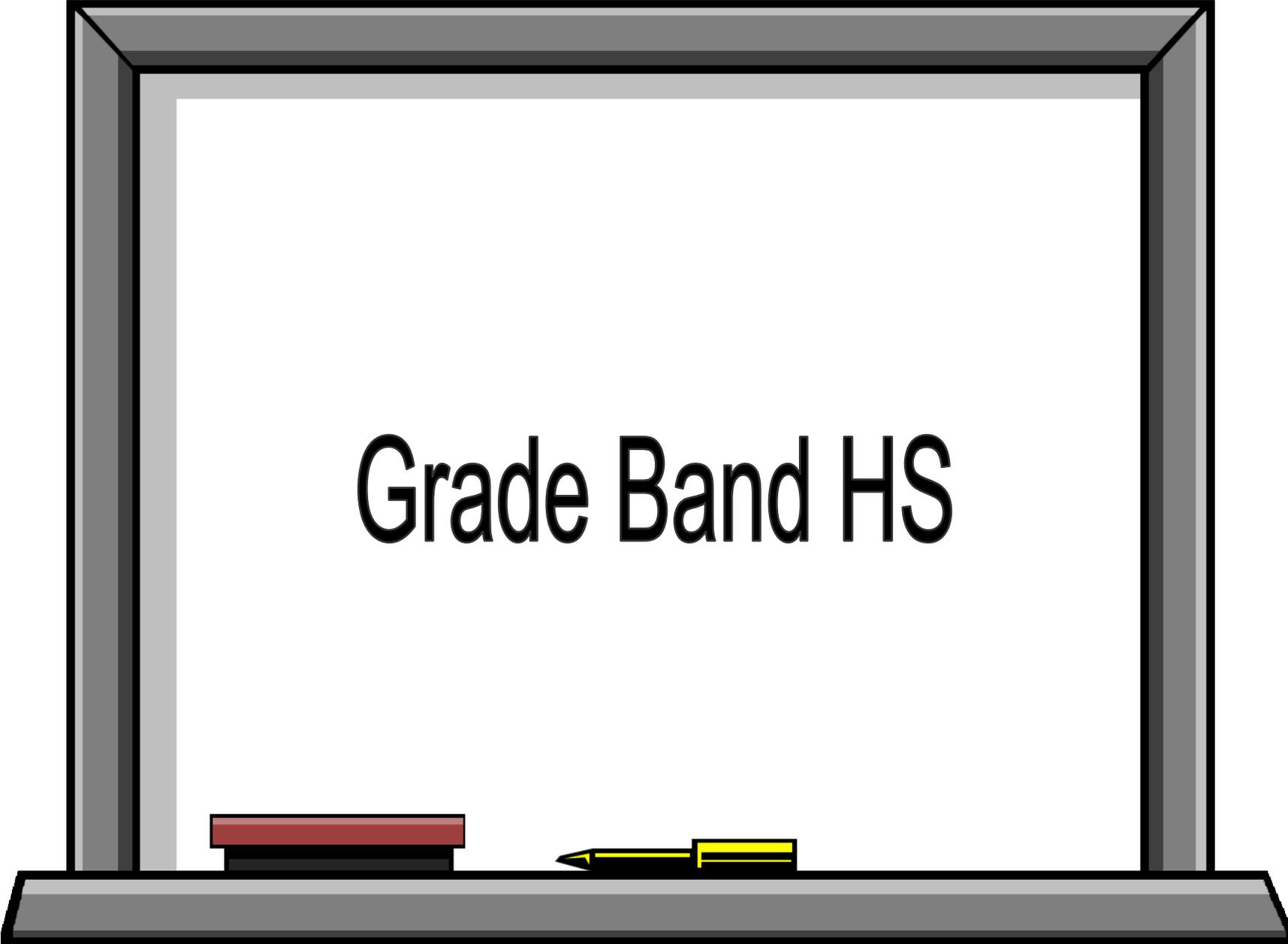
- 8.6 Explain a proof of the Pythagorean Theorem and its converse.  
**Essence: Pythagorean Theorem is a formula that only applies to right triangles**  
 E1: Identify the parts of a right triangle (right angle, legs, hypotenuse).  
 E2: Given an assortment of triangles, identify right triangles.  
 E3: Given shapes, identify triangles.
- 8.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.  
**Essence: Pythagorean Theorem**  
 E1: Identify the parts of a right triangle (right angle, legs, hypotenuse).  
 E2: Identify right triangles in the environment.  
 E3: Identify triangles in the environment.
- 8.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.  
**Essence: Pythagorean Theorem**  
 E1: Given three points on a coordinate plane, determine if the points create a right triangle.  
 E2: Identify a right triangle when drawn on a coordinate plane.  
 E3: Given shapes, identify triangles.

**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**

- 8.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.  
**Essence: Volume of cones, cylinders, and spheres**  
 E1: Label cones, cylinders, and spheres.  
 E2: Find cones, cylinders, and spheres in the environment.  
 E3: Sort three-dimensional shapes (cones, cylinders, spheres).

## Investigate patterns of association in bivariate data.

- 8.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.  
**Essence: Describe patterns on a graph**  
 E1: Identify patterns on scatter plots as positive, negative, or no association.  
 E2: Identify an outlier or cluster within a pattern on a scatter plot.  
 E3: Follow a simple pattern.
- 8.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.  
**Essence: Describe patterns on a graph using a line of best fit**  
 E1: Create a line of best fit when given a scatter plot.  
 E2: Given a scatter plot and multiple lines, select which line most closely represents the line of best fit.  
 E3: Construct a line to connect two points on a graph.
- 8.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*  
**Essence: Describe patterns on a graph using slope and intercept**  
 E1: Given a graph with a line, describe the line as positive, negative, or no association.  
 E2: Match representations of positive, negative, and no association to sample graphs.  
 E3: Identify directionality of lines (going up, going down).
- 8.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*  
**Essence: Construct a two-way table and interpret association between the two variables**  
 E1: Given a two-way table, determine association between two variables.  
 E2: Match representations of positive, negative, and no association to sample graphs.  
 E3: Identify directionality of lines (going up, going down).



Grade Band HS



## Interpret the structure of expressions.

HS.1 Interpret expressions that represent a quantity in terms of its context.

**Essence: Interpret expressions**

- E1: Identify how many terms there are in the equation.
- E2: Identify expressions as numeric or variable.
- E3: Identify mathematical symbols (+, -, =).

HS.2 Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .*

**Essence: Write equivalent expressions**

- E1: Rewrite linear equations to show equivalence ( $2x + 3 = 3 + 2x$ ).
- E2: Rewrite expressions with whole numbers to show equivalence ( $7 + 5 = 5 + 7$ ).
- E3: Use a visual representation or manipulatives to show equivalent expressions with whole numbers less than 10.

## Write expressions in equivalent forms to solve problems.

HS.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

**Essence: Use factoring to create equivalent expressions**

- E1: Match given examples of equivalent expressions using factoring ( $2(x + 4) = 2x + 8$ ).
- E2: Identify equivalent expressions with whole numbers ( $7 + 5 = 5 + 7$ ).
- E3: Use a visual representation or manipulatives to show equivalent expressions with whole numbers less than 10.

### Create equations that describe numbers or relationships.

- HS.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- Essence: Create equations to solve problems with one variable**
- E1: Given a problem that requires a linear equation, select the best equation out of a group of choices to solve.
- E2: Given a one-step problem, identify which operation (addition, subtraction, multiplication, division) is needed to solve.
- E3: Use a visual representation or manipulatives to solve a one-step problem involving whole numbers less than 10.
- HS.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Essence: Create equations and graph on a coordinate axis**
- E1: Match the equation that corresponds to the appropriate graph and then label the axes.
- E2: Match the equation that corresponds to the appropriate graph.
- E3: Identify the x- and y-axis on the graph.
- HS.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .*
- Essence: Rearrange an equation to highlight a specific variable**
- E1: Given a one-step formula, rearrange for the highlighted variable.
- E2: Given a formula, identify which operation would be used to solve for the highlighted variable (the inverse operation).
- E3: Identify mathematical symbols (+, -, =).

**Understand solving equations as a process of reasoning and explain the reasoning.**

- HS.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**Essence: Justify a solution method**

E1: Given the steps to solve an equation, sequence the steps in the correct order.

E2: Identify operation(s) needed to solve an equation.

E3: Match the equation with an equivalent expression.

- HS.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

**Essence: Extraneous solutions may arise when solving simple equations**

E1: Given an equation that models a real-world problem and potential solutions, identify between viable and nonviable solutions.

E2: Given a real-world problem and two solutions, identify which one is viable.

E3: Given a number sentence involving addition or subtraction of numbers less than 10 and two solutions, identify which solution is viable.

**Solve equations and inequalities in one variable.**

- HS.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**Essence: Solve linear equations with coefficients represented by letters**

E1: Solve a one-step linear equation including coefficients represented by letters ( $bx = 20$ ).

E2: Solve a one-step linear equation by using addition or subtraction including a constant represented by a letter ( $x + b = 20$ ).

E3: Solve for the missing number within a given number sentence involving addition or subtraction of numbers less than 10.

**Solve systems of equations.**

- HS.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Essence: Solve a system of linear equations with graphs**

E1: Given one graphed line and two points, connect the points to create a second line and identify the coordinates of the point of intersection.

E2: Given a graph with two intersecting lines, identify the point of intersection and state the coordinates of that point.

E3: Given a graph with two intersecting lines, identify the point of intersection.

- HS.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .*

**Essence: Graph and solve a system with a linear relationship and a quadratic relationship**

E1: Given the graphed quadratic and two points, connect the points to create a line and identify the coordinates of the point(s) where the quadratic and line intersect.

E2: Given a graph of a quadratic and a line, identify the coordinates of the point(s) where the quadratic and line intersect.

E3: Given a graph of a quadratic and a line, identify where the quadratic and line intersect.

### Experiment with transformations in the plane.

HS.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

**Essence: Demonstrate rotations (turns), reflections (flips), and translations (slides)**

E1: Demonstrate a rotation, reflection, or translation.

E2: Manipulate multiple shapes to fit matching spaces.

E3: Match shapes in different orientations.

### Understand similarity in terms of similarity transformations.

HS.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**Essence: Identify similar figures**

E1: Categorize triangles by more than one attribute (acute, obtuse, right angles) and label attributes.

E2: Categorize shapes by more than one attribute and label attributes.

E3: Match shapes by more than one attribute.

HS.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

**Essence: Pythagorean Theorem is a formula that only applies to right triangles**

E1: Construct a right triangle on a coordinate plane and label the parts.

E2: Identify the parts of a right triangle (right angle, legs, hypotenuse).

E3: Given an assortment of triangles, identify right triangles.

## Geometry—Circles (C)

HS

Understand and apply theorems about circles.

HS.1 Prove that all circles are similar.

**Essence:** *Identify the properties of circles*

E1: Describe how circles are similar.

E2: Identify three-dimensional shapes with a circle as a cross-section.

E3: Sort circles from other shapes.

### Visualize relationships between two-dimensional and three-dimensional objects.

HS.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

***Essence: Identify the shapes of cross-sections of three-dimensional objects***

E1: Label shapes of cross-sections of three-dimensional objects.

E2: Categorize three-dimensional objects by the shape of their cross-sections.

E3: Sort three-dimensional objects (cones, cylinders, spheres).

### Apply geometric concepts in modeling situations.

HS.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

***Essence: Identify geometric shapes in the real world***

E1: Label real-world objects as particular geometric shapes.

E2: Find cones, cylinders, spheres, pyramids, or cubes in the environment.

E3: Match cones, cylinders, or spheres with real-world objects.

**Summarize, represent, and interpret data on a single count or measurement variable.**

HS.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

***Essence: Represent data with plots***

E1: Create a histogram to represent given data.

E2: Given a plot including axes and labels, create the bars to complete a histogram.

E3: Locate specified points on a number line up to 20.

HS.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

***Essence: Use measures of center to compare data***

E1: Given a data set involving numbers less than 100, compute mean (average), median, or mode.

E2: Given a data set involving numbers less than 100, select the appropriate mean (average), median, or mode from given options.

E3: Given a data set involving numbers less than 20, select the appropriate mean (average) from given options.

**Summarize, represent, and interpret data on two categorical and quantitative variables.**

HS.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

***Essence: Interpret categorical data in a two-way frequency table***

E1: Given a two-way table, determine association between two variables.

E2: Determine the rule for a numeric pattern and extend.

E3: Extend a simple numeric pattern.

HS.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

***Essence: Interpret data on a scatter plot***

E1: Describe the scatter plot as positive, negative, or no association.

E2: Given a scatter plot and multiple lines, select which line most closely represents the line of best fit.

E3: Construct a line to connect two points on a graph.

**Interpret linear models.**

HS.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

***Essence: Interpret the slope and intercept on a graph***

E1: Given a graph with a line, identify the slope from given options.

E2: Given a graph with a line, describe the line as positive, negative, or no association.

E3: Match representations of positive, negative, or no association to sample graphs.

### Understand and evaluate random processes underlying statistical experiments.

HS.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

**Essence: Evaluate if data is consistent with a data-generating process**

E1: Given data and a data-generating device (spinner, coin, dice), determine if the data could come from the specified device.

E2: Given a data-generating device, determine the probability (likely, impossible) of different outcomes.

E3: Match data from the data-generating device to categories corresponding to the device.

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